

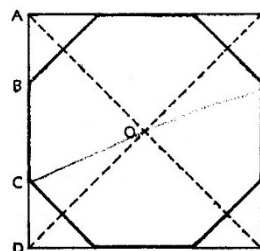
SCHOLARSHIP EXAMINATION, 1942.

Mathematics II.

(2 hours).

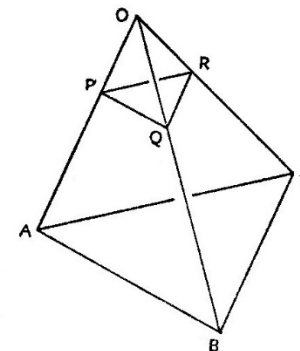
(Questions may be done in any order : read through the paper first).

- Multiply $(a-b)$ by $(c+d)$. Use your answer to find $15\frac{1}{2} \times 38.11d$.
A well-known approximation to π is $3 + \frac{1}{7} (1 - \frac{1}{118})$. Evaluate this to 7 places of decimals. To how many significant figures does it agree with the true value of π , i.e. 3.1415926535 ?
- $I(x)$ means the largest whole number not greater than x . For example $I(\frac{1}{4})=0$, $I(8\frac{1}{2})=8$, $I(0.2)=0$, $I(-3.7)=-4$. Evaluate $I(2x)-2.I(x)$ for $x=3, 3.1, 3.2$, etc. up to 4. What can you say about x if $I(2x)-2.I(x)=0$? What values can $I(x+y)-I(x)-I(y)$ have? What values can $I(nx)-n.I(x)$ have, if n is a positive whole number?
- The figure shows a regular octagon inscribed in a square. Find the angles of the triangle ACO , and prove that $AC=AO$. Calculate BC if $AD=4$ inches.



- 20 cubic inches of water are poured into a hollow inverted cone whose angle at the vertex is 70° . How deep will the water be in the centre, and what will be the area of its top surface?
(The volume of a cone of base radius r and height h is $\frac{1}{3}\pi r^2 h$).

- $OABC$ is a triangular pyramid. $OP=\frac{1}{4}.OA$, $OQ=\frac{1}{4}.OB$, $OR=\frac{1}{4}.OC$.
Prove that (i) the triangles OAB , OPQ are similar.
(ii) the triangles ABC , PQR are similar.
What fraction of the volume of the pyramid is the volume of $OPQR$?



- Two boys buy bicycles with 26-inch and 28-inch diameter wheels respectively. They also buy appropriate cyclometers, but by mistake they are fitted to the wrong machines. They go for a ride and find the cyclometer readings differ by 2.7 miles. How far did they ride?
(The numerical value of π is not needed. Think what the readings are after one mile).
- (i) A vessel which can hold $a+b+c$ pints contains $a+b$ pints of milk. It is filled up from another vessel containing a pints of milk and b pints of water thoroughly mixed. How much milk and how much water will there now be in the first vessel? ($a+b > c$).
(ii) There are ten pint measures, $A, B, C, \dots, J$. A is half full of milk, the rest are half full of water. The contents of A are poured into B and well mixed; half a pint of the mixture is then poured back into A . The contents of B are then poured into C , and half a pint of the mixture poured back into B . This is repeated to the end of the row. What is the proportion of milk to water in J ?

[TURN OVER