# SHERBORNE SCHOOL.

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ENTRANCE SCHOLARSHIP EXAMINATION, 1954.

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## Mathematics II.

(2 hours).

Questions may be answered in any order. A few questions carefully answered are better than many scraps.

1. A shopkeeper buys a camera for £5, to which has to be added a fixed percentage for Purchase Tax. He then adds the same percentage of the new total as his own profit. Find this percentage if the final price is £9. 16s. If the Purchase Tax falls to 25%, what percentage profit will he make if he sells a similar camera for £9?

2. $A$ and $B$ are two points on the same side of a straight line $XY$. Give a geometrical construction for the shortest route from $A$ to some point on $XY$ and on to $B$. Justify your construction.

   The locus of a point $P$ for which the sum of distances $PA + PB$ from two fixed points $A$ and $B$ is constant, is a closed oval curve called an *ellipse*. Sketch the ellipse roughly when $AB = 2$ inches, $PA + PB = 3$ inches. Obviously $PA + PB$ is more than 3 ins. for a point $P$ outside the ellipse. What does the result of the construction above tell you about the angles between $PA$, $PB$ and the tangent to the ellipse at $P$?

3. Write down three consecutive whole numbers, the middle one of which is $n$. If $a$, $b$, $c$ are three consecutive whole numbers, prove that $a^2 - 2b^2 + c^2$ is always the same, and find its value.

   Look at these statements: $3^3 = 1^3 + 6 \times 2^2 + 2$

   $5^3 = 3^3 + 6 \times 4^2 + 2$

   $7^3 = 5^3 + \dots\dots\dots$

   Complete the last line: verify it, and make a general statement beginning $(n+1)^3 = \dots\dots$ Prove your statement.

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4. $ABCD$ is a rectangle in which $AB = 20$ ft., $AD = 5$ ft. $AB$ is produced to $E$ making $BE = 15$ ft., and $BC$ is produced to $F$ making $CF = 15$ ft. Prove that $EF$ and $FD$ are both 25 ft., and that the angle $EFD$ is a right angle. The quadrilateral $AEFD$ is the plan of a room which is to be covered with tiles, each 1 ft. square. No tiles are to be cut except at the edges of the room. Draw the figure on squared paper, and show how it is possible to lay the tiles, beginning with a tile in corner $A$, and cutting a certain number of tiles into two pieces, both of which can be used. Find how many tiles will be needed, and how many will have to be cut.

5. A man goes to a drawer full of socks in the dark.

   (i) There are black socks and blue socks in it, but he cannot tell the difference. How many socks must he take to be sure of having a pair?

   (ii) There are four kinds of socks in the drawer, and he wants six all alike. How many must he take now?

   (iii) There are $p$ kinds of socks in the drawer and he wants $q$ socks all alike. How many must he take?

6. An inkwell is a cube of glass 4 inches each way. It has a hemispherical bowl hollowed out for the ink, 2 inches in diameter. Also it has four feet, which are hemispherical bosses 1 inch in diameter, on its base, and four exactly similar bosses on the top. Find the weight of it, if 1 cu. in. of glass weighs $1\frac{1}{4}$ oz. (You are warned that the value of $\pi$ will not be wanted).

7. $ABC$ is a triangle, right-angled at $A$. $AN$ is drawn perpendicular to $BC$ to meet it at $N$. Name three similar triangles in the figure, with the vertices in the correct corresponding order.
   Hence show that $BN.BC = BA^2$ and $CN.CB = CA^2$. What result do you get if you add these two equations? If you draw the actual squares on $AB$, $AC$ and $BC$, and produce $AN$, can you illustrate these equations in terms of areas?