SHERBORNE SCHOOL.

SUMMER EXAMINATION, 1898.

Army Class.

General Paper.

1. A railway train is travelling along a curve of $\frac{1}{2}$ mile radius at the rate of 25 miles per hour: through what angle will it turn in half a minute?

2. Prove the following identities:

(1) $\cos 2A = \dfrac{\cot^2 A - 1}{\cot^2 A + 1}$;

(2) $\dfrac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} = \tan\dfrac{\theta}{2}$;

(3) $\dfrac{\cos 3A + \sin 3A}{\cos A - \sin A} = 1 + 2\sin 2A$.

3. Find $\sin 18°$. If $A + B + C = 180°$, prove

(1) $\tan\dfrac{A}{2}\cdot\tan\dfrac{B}{2} + \tan\dfrac{B}{2}\cdot\tan\dfrac{C}{2} + \tan\dfrac{C}{2}\cdot\tan\dfrac{A}{2} = 1$;

(2) $\tan\dfrac{A-B}{2} = \dfrac{a-b}{a+b}\cdot\cot\dfrac{C}{2}$;

(3) $\dfrac{b^2-c^2}{a}\cos A + \dfrac{c^2-a^2}{b}\cos B + \dfrac{a^2-b^2}{c}\cos C = 0$.

4. Find in three different forms a formula for the area of a triangle.

Find the area of the triangle in the following cases:

(1) $a = 171$, $b = 204$, $c = 195$;

(2) $a = 2(\sqrt{3}+1)$ inches, $B = 45°$, $C = 60°$;

(3) $a = b = 10$, $C = 30°$.

5. Find the values of R, r, r_1 in terms of the sides of the triangle. Prove

(1) $r = 4R \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$;

(2) $r_1 + r_2 = c \cot\frac{C}{2}$.

6. Find a general formula for all the angles which have a given cosine.

Solve

(1) $\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$;

(2) $\tan^2\theta = \frac{1}{3}$;

(3) $\sin\theta + \sqrt{3}\cos\theta = \sqrt{2}$.

7. Solve the equations:

(1) $x^5 - 3x^4 - 7x^3 - 7x^2 - 3x + 1 = 0$;

(2) $(x+1)^2 = x + 3\sqrt{3x^2 + 3x - 11}$;

(3) $\sqrt{3x+7} + \sqrt{2x+3} = 7$;

(4) $\left.\begin{aligned} x^3 + 2y^3 &= 10 \\ x + y &= 3 \end{aligned}\right\}$.

8. Show that, if $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$, then each fraction is equal to $\frac{1}{2}$ or -1.

9. If b is half the harmonic mean between a and c prove that

$$a^3 - b^3 + c^3 + 3abc = (a - b + c)^3.$$

10. Simplify

$$\frac{3}{2(x-1)} + \frac{1}{2(x+1)} + \frac{x-2}{x^2+1} + \frac{2(x^3+2)}{1-x^4}.$$

11. In a given straight line find a point such that the sum of its distances from two given points is a minimum.

12. Find the condition that a quadrilateral may be described about a circle.